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Off-shell Supersymmetry versus Hermiticity in the Superstring

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We point out that off-shell four-dimensional spacetime-supersymmetry implies strange hermiticity properties for the N=1 RNS superstring. However, these hermiticity properties become natural when the N=1 superstring is embedded into an N=2 superstring.

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In four-dimensional compactifications of the N=1 RNS superstring, the spacetime-supersymmetry generators in the $-\frac{1}{2}$ picture are

$$q_a = \frac{1}{2\pi i}\oint dz\ e^{\frac{1}{2}(-\phi \pm i\sigma_0 \pm i\sigma_1 \pm iH_C)} \tag{1}$$

where there are an even number of + signs in the exponential ($a = 1$ to 4), ϕ comes from fermionizing the bosonic ghosts as $\beta = i\partial\xi e^{-\phi}$ and $\gamma = -i\eta e^{\phi}$, $\psi^3 \pm \psi^0 = e^{\pm i\sigma_0}$ and $\psi^1 \pm i\psi^2 = e^{\pm i\sigma_1}$ where ψ^m is the fermionic vector, and $\partial H_C = J_C$ is the U(1) generator of the c=9 N=2 superconformal field theory representing the compactification manifold.

These spacetime-supersymmetry generators satisfy the anti-commutation relations

$$\{q_a, q_b\} = \frac{1}{2\pi i}\oint dz\ e^{-\phi}\psi_m\gamma^m_{ab} \tag{2}$$

which is not the usual supersymmetry algebra $\{q_a, q_b\} = \frac{1}{2\pi}\oint dz\ \ \partial x_m\gamma^m_{ab}$ where $\frac{1}{2\pi}\oint dz\ \ \partial x_m$ is the string momentum. However after hitting the right-hand side of (2)with the picture-changing operator $Z = \{Q, \xi\}$, it becomes $\frac{1}{2\pi i}\oint dz\ \ Ze^{-\phi}\psi_m\gamma^m_{ab} = \frac{1}{2\pi}\oint dz\ \partial x_m\gamma^m_{ab}$. So up to picture-changing, the q_a's form a supersymmetry algebra.[1]

But off-shell supersymmetry requires that the q_a's form a supersymmetry algebra without applying picture-changing operations. This is because picture-changing is only well-defined when the states are on-shell. Off-shell, the states are not independent of the locations of the picture-changing operators.

So off-shell spacetime-supersymmetry requires modification of the q_a's. Note that q_a has picture $-\frac{1}{2}$ and the momentum $\frac{1}{2\pi}\oint dz\ \partial x_m$ has picture 0, so we need generators with picture $+\frac{1}{2}$. The obvious solution[2] is to split q_a into a chiral part with picture $-\frac{1}{2}$ and an anti-chiral part with picture $+\frac{1}{2}$:

$$q_\alpha = \frac{1}{2\pi i}\oint dz\ e^{\frac{1}{2}(-\phi \pm i(\sigma_0+\sigma_1) + iH_C)} \tag{3}$$

$$\bar{q}_{\dot\alpha} = Zq_{\dot\alpha} = \frac{1}{2\pi i}\oint dz\ [b\eta e^{\frac{1}{2}(3\phi \pm i(\sigma_0-\sigma_1) - iH_C)}$$
$$+i\ \ :(e^{\phi}\psi_m\partial x^m + e^{\phi}G^+_C + e^{\phi}G^-_C)e^{\frac{1}{2}(-\phi \pm i(\sigma_0-\sigma_1) - iH_C)}\ :]$$

where $G^{\pm}_C$ are the fermionic generators of the c=9 N=2 superconformal field theory. The N=1 4D supersymmetry algebra $\{q_\alpha, \bar{q}_{\dot\beta}\} = \frac{1}{2\pi}\oint dz\ \partial x_m\sigma^m_{\alpha\dot\beta}$ is now satisfied off-shell where we are using standard two-component Weyl notation.

Although we have solved the problem of finding off-shell supersymmetry generators, we now have a new problem. Using the standard RNS definition of hermiticity where all fundamental fields are hermitian or anti-hermitian (the anti-hermitian field is σ_0), the hermitian conjugate of q_α is no longer $\bar{q}_{\dot\alpha}$. Fortunately, this new problem can be solved by modifying the definition of hermiticity. However, this new hermiticity definition will only be natural if one embeds the N=1 superstring into an N=2 superstring.

To find the appropriate hermiticity definition, one first writes $\bar{q}_{\dot\alpha}$ in the form

$$\bar{q}_{\dot\alpha} = e^R \ (\frac{1}{2\pi i}\oint dz \ b\eta e^{\frac{1}{2}(3\phi \pm i(\sigma_0-\sigma_1)-iH_C)}) \ e^{-R}$$

where

$$R = \frac{1}{2\pi}\oint dz \ c\xi e^{-\phi}(\psi^m \partial x_m + G_C^+ + G_C^-) \tag{4}$$

and $e^R F e^{-R} = F + [R,F] + \frac{1}{2}[R,[R,F]] + ...$ (the expansion usually stops after two terms).

One then defines hermiticity as:

$$(x_m)^\dagger = e^R x_m e^{-R}, \quad (\psi_m)^\dagger = e^R \psi_m e^{-R}, \quad ,(F_C)^\dagger = e^R \bar{F}_C e^{-R}, \tag{5}$$

$$(e^{\frac{\phi}{2}})^\dagger = e^R(c\xi e^{-\frac{3}{2}\phi})e^{-R}, \quad (e^{-\frac{\phi}{2}})^\dagger = e^R(b\eta e^{\frac{3}{2}\phi})e^{-R},$$

$$(b)^\dagger = e^R(ib\eta\partial\eta e^{2\phi})e^{-R}, \quad (c)^\dagger = e^R(-ic\xi\partial\xi e^{-2\phi})e^{-R},$$

$$(\eta)^\dagger = e^R(i\eta b\partial b e^{2\phi})e^{-R}, \quad (\xi)^\dagger = e^R(-i\xi c\partial c e^{-2\phi})e^{-R},$$

where F_C are the worldsheet fields in the c=9 N=2 superconformal field theory. It is straightforward to check that the new hermiticity definition satisfies $(F^\dagger)^\dagger = F$ for all F, preserves OPE's, and implies that $(q_\alpha)^\dagger = \bar{q}_{\dot\alpha}$.

One strange feature of the hermiticity definition of (5)is that a field may have a different conformal weight from its hermitian conjugate since $(T)^\dagger = T + i\partial(bc+\xi\eta)$ where T is the RNS Virasoro generator. Another strange feature is that the BRST operator is not hermitian since $Q^\dagger = \frac{1}{2\pi}\oint dz \ b$. (This easily follows from writing $Q = e^R(\frac{i}{2\pi}\oint dz \ b\eta\partial\eta e^{2\phi})e^{-R}$.)

Although these features are strange in the N=1 RNS description of the superstring, they are natural if the N=1 superstring is embedded into an N=2 superstring. As discussed in reference [3], any critical N=1 superstring can be embedded into a critical N=2 superstring where the c=6 N=2 superconformal generators are[3]

$$T_{N=2} = T_{N=1} + \frac{i}{2}\partial(bc+\xi\eta), \quad G^+_{N=2} = j_{BRST}, \quad G^-_{N=2} = b, \quad J_{N=2} = bc+\xi\eta, \tag{6}$$

and $j_{BRST} = e^{R}(ib\eta\partial\eta e^{2\phi})e^{-R}$.

Using the hermiticity definition of (5), $(T_{N=2})^{\dagger} = T_{N=2}$, $(G^{+}_{N=2})^{\dagger} = G^{-}_{N=2}$, and $(J_{N=2})^{\dagger} = J_{N=2}$, which are the standard hermiticity properties of an N=2 string. (This hermiticity can be made manifest by writing the N=2 generators of (6)in terms of spacetime-supersymmetric variables.[2]) So the hermiticity properties implied by off-shell four-dimensional supersymmetry are natural only if the N=1 superstring is embedded into an N=2 superstring.

References

[1] D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. B271 (1986) p.93.

[2] N. Berkovits, Nucl. Phys. B431 (1994) p.258.

[3] N. Berkovits and C. Vafa, Mod. Phys. Lett. A9 (1994) p.653.